Equal sums of four n th power's for (n=2,3 &4)

$$a^n + b^n + c^n + d^n = e^n + f^n + g^n + h^n$$

For n=2,

Parametric form:

$$a^2 + b^2 + c^2 + d^2 = e^2 + f^2 + g^2 + h^2$$

We have,

Identity: $(p^2 + q^2)^2 = (p^2)^2 + (q^2)^2 + (pq)^2 + (pq)^2 = (a, b, c, d)^2 \;--(1)$

$$(m^2 + n^2)^2 = (m^2)^2 + (n^2)^2 + (mn)^2 + (mn)^2 = (e, f, g, h)^2 --(2)$$

Since equation, $(p^2 + q^2 = m^2 + n^2)$ can be parameterized as below:

$$(p, q, m, n) = [(6m^2), (8m^2 - 1), (2m)(10m^2 - 1)]$$

Hence, equation (1) =(2)

And, $(a, b, c, d)^2 = (e, f, g, h)^2$

Where, $(a, b, c, d) = [(36m^4), (8m^2 - 1)^2, (48m^4 - 6m^2), (48m^4 - 6m^2)]$

And, $(e, f, g, h) = [(4m^2), (10m^2 - 1)^2, (20m^3 - 2m), (20m^3 - 2m)]$

For, (m,n)=(2,1) we get, (2-4-4) equation. Namely four suares equal to another four squares. Two chains Taxicab.

$$(a, b, c, d) = (\,576, 961, 744, 744\,)$$

$$(e, f, g, h) = (\,16, 1521, 156, 156\,)$$

$$(\,576, 961, 744, 744\,)^2 = (\,16, 1521, 156, 156\,)^2$$

Solution for taxicab three chains is given below:

$$a^2 + b^2 + c^2 + d^2 = e^2 + f^2 + g^2 + h^2 = i^2 + j^2 + k^2 + l^2$$

$$(p^2 + q^2)^2 = (p^2)^2 + (q^2)^2 + (pq)^2 + (pq)^2 = (a, b, c, d)^2 \quad --(1)$$

$$(m^2 + n^2)^2 = (m^2)^2 + (n^2)^2 + (mn)^2 + (mn)^2 = (e, f, g, h)^2 --(2)$$

$$(u^2 + v^2)^2 = (u^2)^2 + (v^2)^2 + (uv)^2 + (uv)^2 = (I, j, k, l)^2 --(3)$$

We have parametric solution for eqn. (1), (2)&(3):

$$p = (13x^2 - 182xy - 13y^2)$$

$$q = (91x^2 + 26xy - 91y^2)$$

$$m = (89x^2 - 46xy - 89y^2)$$

$$n = (23x^2 + 178xy - 23y^2)$$

$$u = (85x^2 - 70xy - 85y^2)$$

$$v = (35x^2 - 170xy - 35y^2)$$

For (x,y)=(1,1) we get:

$$(13,91)^2 = (23,89)^2 = (35,85)^2$$

Hence (p,q,m,n,u,v)=(13,91,23,89,35,85)

After substituting values of (p,q,m,n,u,v) in equation's (1), (2) &(3) we get:

$$(169,8281,1183,1183)^2 = (529,7921,2047,2047)^2 =$$

$$(1225,7225,2975,2975)^2$$

Above is Taxicab three chain equation. (2-4-4) equation in three ways.

Note: Parametric solution is also possible for taxicab of four chain.

For n=3,

$$a^3+b^3+c^3+d^3=e^3+f^3+g^3+h^3$$

Parametric form:

We have,

Identity: $(p^3+q^3)^2 = (p^2)^3+(q^2)^3+(pq)^3+(pq)^3=(a,b,c,d)^3 \;--(1)$

& $(m^3+n^3)^2 = (m^2)^3+(n^2)^3+(mn)^3+(mn)^3=(e,f,g,h)^3--(2)$

Since, equation, $(p^3+q^3=m^3+n^3)$ can be parameterized as below:

(p,q,m,n)=$[(a^2-7a-9),(2a^2-4a+12),(2a^2+10)(a^2-9a-1)]$

Hence, equation (1) =(2)

And, $(a,b,c,d)^3=(e,f,g,h)^3$

Where, (a,b,c,d)=

$[(a^2+7a-9)^2,(2a^2-4a+12)^2,$

$((a^2+7a-9)*(2a^2-4a+12)),((a^2+7a-9)*(2a^2-4a+12))]$

And, $(e,f,g,h)=$

$[(2a^2+10)^2,(a^2-9a-1)^2,$

$((2a^2+10)*(a^2-9a-1)),((2a^2+10)*(a^2-9a-1))]$

For, (a)=(1) we get:

$$(a,b,c,d)=(1,100,108,108)$$

$$(e,f,g,h)=(10,10,81,144)$$

$$(1,100,108,108)^3 = (10,10, 81,144)^3$$

For n=4,

$$a^4 + b^4 + c^4 + d^4 = e^4 + f^4 + g^4 + h^4$$

Parametric form:

We have,

Identity: $(p^4 + q^4)^2 = (p^2)^4 + (q^2)^4 + (pq)^4 + (pq)^4 = (a, b, c, d)^4 \ --(1)$

Also, $(m^4 + n^4)^2 = (m^2)^4 + (n^2)^4 + (mn)^4 + (mn)^4 = (e, f, g, h)^4 --(2)$

Since equation, $(p^4 + q^4 = m^4 + n^4)$ can be parameterized as below:

(p,q,m,n) $= [(a + 3a^2 - 2a^3 + a^5 + a^7), (1 + a^2 - 2a^4 - 3a^5 + a^6),$

$(a - 3a^2 - 2a^3 + a^5 + a^7)$,$(1 + a^2 - 2a^4 + 3a^5 + a^6)$]

Hence, equation (1) =(2)

And, $(a, b, c, d)^4 = (e, f, g, h)^4$

For, (a)=(2) we get, $(p, q, m, n) = (158,59,134,133)$

And, $(a, b, c, d) = (24964, 3481, 9322, 9322)$

$(e, f, g, h) = (17956,17689,17822,17822)$

$$(24964, 3481, 9322, 9322)^4 \quad =$$

$$(17956,17689,17822,17822)^4$$

Note: A (2-3-3) equation can be arrived at by using the below Identity:

$$a^4 + b^4 = (a^2 - b^2)^2 + (ab)^2 + (ab)^2$$

$$c^4 + d^4 = (c^2 - d^2)^2 + (cd)^2 + (cd)^2$$

We know, (a^4+b^4)=(c^4+d^4) for (a,b,c,d)=(134,133,158,59)

Hence we get, $(267,17822,178220)^\wedge 2 = (21483,9322,9322)^\wedge 2$
