Sum difference of four fifth powers = four square's

Consider the below equation:

$$(a^5 + b^5 + c^5 + d^5) = (p^2 + q^2 + r^2 + s^2) - - - - -(1)$$

Above equation (1) has numerical solution given below:

$$(2, 23, -3, -22)^5 = (2, 3, 4, 22)^2$$

We have below the well-known identity:

$$(a + b + c)^5 - (a^5 + b^5 + c^5) \quad = 5(a + b) * (b + c) *$$

$$(c + a) * [(a^2 + b^2 + c^2) + (ab + bc + ca)] \text{ ---------- (1)}$$

In the above identity, we first parameterize $(ab + bc + ca)$ on the right hand side to make it a square, say$(v)^2$. Then the parameter's (a, b, c) we substitute in $[5(a + b)(b + c)(c + a)]$ & we make this also a square say(u^2). The result is that we will have a right hand side (RHS) which is given below.

RHS= $u^2(a^2 + b^2 + c^2 + v^2) = [(ua)^2 + (ub)^2 + (uc)^2 + (uv)^2]$

Above which is a sum of four squares.

For brevity we call $(a + b + c) = p$

So, now we have a new equation from equation (1):

$$[(p)^5 - (a)^5 - (b)^5 - (c)^5)] =$$

$$[(ua)^2 + (ub)^2 + (uc)^2 + (uv)^2]$$

Above can also be written as:

$$[(p), (-a), (-b), (-c)]^5 = [ua, ub, uc, uv]^2 \; - - - -(2)$$

Now we have numerical solution for $[(ab+bc+ca)=(v^2)]$ as:

$$(a,b,c,v)=(\,2,2,3,4\,)$$

From the above we get the parametric form;

$$(a,b,c,v)=((-mn),(5m^2+mn),(n^2+mn),(2mn))$$

Next we substitute the values of (a,b,c) in $[5(a+b)(b+c)(c+a)]$ & we get the value:

$$=(5mn)^2*(5m^2+2mn+n^2)$$

Above is a square if $(5m^2+2mn+n^2)$ is a square say (w^2).

Now $(5m^2+2mn+n^2)=w^2$ has numerical solution

$$(\,m,n,w)=\;(1,-1,\;2)$$

Hence after parameterization we get:

$$(m,n,w)=\left((3k^2+2k),(5k^2+10k+4),(10k^2+12k+4)\right)$$

Hence we have:

$$v=2mn=2(3k^2+2k)(5k^2+10k+4)$$

$$u=\;5mnw$$

$$=5(3k^2+2k)(5k^2+10k+4)(10k^2+12k+4)$$

$$a=-(3k^2+2k)(5k^2+10k+4)$$

$$b=4k(3k+2)(5k^2+5k+1)$$

$$c=4(k+1)(2k+1)(\,5k^2+10k+4)$$

$$p=[4k^4+k^2(2k-1)+(9k^2+11k+4)^2]$$

So, substituting for (a, b, c, v, w) in equation (2) for $(k = 1)$ we get:

$$[(p), (-a), (-b), (-c)]^5 = [ua, ub, uc, uv]^2$$

$$(581, 95, -220, -456)^5 = (26)^2(475)^2(95{,}220{,}456{,}190)^2$$

Which is sum difference of four fifth powers equal to four squares.
