Equation: $(\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d})^{\boldsymbol{n}}=\boldsymbol{n}(\boldsymbol{e})^{\mathbf{2}}$

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$$\boldsymbol{a}^{\boldsymbol{n}}+\boldsymbol{b}^{\boldsymbol{n}}+\boldsymbol{c}^{\boldsymbol{n}}+\boldsymbol{d}^{\boldsymbol{n}}=\boldsymbol{n}(\boldsymbol{e})^{\mathbf{2}} \text{--------} (1)$$

Parametric solutions are given below for degree, $n=2,3,4$ & 5

<u>For, n=2</u>

$$(a,b,c,d)^2=2(e)^2$$

$$a=4m^2+6mn-4n^2$$

$$b=4m^2-6mn-4n^2$$

$$c=3m^2+8mn-3n^2$$

$$d=3m^2-8mn-3n^2$$

$$e=5(m^2+n^2)$$

$$(m,n)=(3,2)\ \ we\ get,\ \ (16,33,56,63)^2=2(65)^2$$

<u>For, n=3</u>

$$(a,b,c,d)^3=3(e)^2$$

$$a=(4p^4+98p^3q+789p^2q^2+2378pq^3+2320q^4)$$

$$b=(5p^4+82p^3q+399p^2q^2+502pq^3-16q^4)$$

$$c=3(p^4+132p^3q+20p^2q^2+320pq^3+256q^4)$$

$$d=3(p^4+132p^3q+20p^2q^2+320pq\text{^}3+256q^4)$$

$$e=(3p^2+30pq+48q^2)^2(p^2+9pq+29q^2)$$

$$(p,q)=(0,1),\qquad we\ get,\qquad (a,b,c,d,e)=(192,580,-4,192,8352)$$

For, n=4

$$(a,b,c,d)^4 = 4(e)^2$$

$$a = (2p-q)^2$$

$$b = (p-2q)^2$$

$$c = (2p-q)(p-2q)$$

$$d = (p-2q)(2p-q)$$

$$e = (17p^4 - 40p^3q + 48p^2q^2 - 40pq^3 + 17q^4)$$

$$(p,q) = (3,1)\ we\ get, \quad (a,b,c,d,e) = (25,5,1,5,313)$$

For, n=5

$$(a,b,c,d)^5 = 5(e)^2$$

$$a = (p^2 - 3pq + 3q^2)$$

$$b = (p-2q)$$

$$c = (q-p)$$

$$d = (-p^2 + 3pq - 2q^2)$$

$$e = (p^4 - 6p^3q + 14p^2q^2 - 15pq^3 + 6q^4)$$

$$(p,q) = (4,1)\ we\ get, \quad (a,b,c,d,e) = (7,-3,2,-6,42)$$

Note: In context to the above article, it is to be noted that, there are no numerical solutions possible for which four sixth powers is equal to six squares. Hence there is no point attempting to parameterize equation (1) above for n=6. But there is parameterization in the form of an identity for which three sixth powers is equal to 12 squares & is given below.

For, $(a+b+c)=0$

$$a^6+b^6+c^6=6(abc)^2+\sum(a^2b)^2$$

$(a,b,c)=(5,-3,-2)$, $(5,3,2)^6=(12,18,20,45,50,75)^2+6(30)^2$
