Taxicab Equations:

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Taxicab parametric equations for 4-2-3 with (Four,Seven & Ten) chains

Taxicab parametric equations for 4-2-3 with (Four) chains

$$Hence\ For\ k = 2,4$$

$$2 * \left((p^2 + pq + q^2)(r^2 + rs + s^2)\right)^k$$

$$= (s^2 + r^2 + rs)^k((p^2 - q^2)^k + (q^2 + 2pq)^k + (-p^2 - 2pq)^k)$$

$$= (p^2 + q^2 + pq)^k((r^2 - s^2)^k + (s^2 + 2rs)^k + (-r^2 - 2rs)^k)$$

$$= \left((2qr + pr + qs - ps)(pr + qs + ps)\right)^k + \left((2qs + ps + 2pr + qr)(ps - qr)\right)^k + \left(-(qs + 2ps + pr - qr)(pr + qr + qs)\right)^k$$

$$= \left(-(2pr + qr + ps - qs)(qr + ps + qs)\right)^k + \left((2qr + pr + 2ps + qs)(pr - qs)\right)^k + \left((ps + 2qs - pr + qr)(ps + pr + qr)\right)^k$$

$$Numerical\ examples:$$

$$For\ (p, q, r, s) = (2,1,3,1)\ we\ get\ the\ four\ chains\ for\ k = 2\ \&\ 4$$

$$2(91)^k = (39,65,104)^k = (49,56,105)^k = (19,80,99)^k = (11,85,96)^k$$

$$For\ (p, q, r, s) = (4,3,5,2)\ , after\ taking\ out\ common\ factors\ we\ get$$

$$2(481)^k = (91,429,520)^k = (259,296,555)^k = (544,175,369)^k = (551,336,215)^k$$

Taxicab parametric equation for 4-2-3 with (seven) chains

For k=2 & 4

$$2[(p^2+pq+q^2)^k(r^2+rs+s^2)^k(3r^2+s^2)^k =$$

$$(r^2+rs+s^2)^k*(3r^2+s^2)^k*((p^2-q^2)^k+(q^2+2pq)^k+(-p^2-2pq)^k) =$$

$$(p^2+pq+q^2)^k*(3r^2+s^2)^k*((r^2-s^2)^k+(s^2+2rs)^k+(-r^2-2rs)^k)=$$

$$(3r^2+s^2)^k*[\big((2qr+pr+qs-ps)(pr+qs+ps)\big)^k+\big((2qs+ps+2pr+qr)(ps-qr)\big)^k+\big(-(qs+2ps+pr-qr)(pr+qr+qs)\big)^k]=$$

$$(3r^2+s^2)^k[\big(-(2pr+qr+ps-qs)(qr+ps+qs)\big)^k+\big((2qr+pr+2ps+qs)(pr-qs)\big)^k+\big((ps+2qs-pr+qr)(ps+pr+qr)\big)^k]=$$

$$(p^2+pq+q^2)^k*(r^2+rs+s^2)*((3r^2+2rs-s^2)^k+(-3r^2+2rs+s^2)^k+(-4r*s)^k)$$

$$=(s^2+r^2+rs)^k*[\big(-(ps-sq+3rp+3rq)(-ps-sq+rp-rq)\big)^k$$

$$+\big(-(2rp+rq-sq)*(2ps+3rq+sq)\big)^k$$

$$+\big((2rq+rp+ps)*(-2sq+3rp-ps)\big)^k$$

$$=(s^2+r^2+rs)^k*[\big((-ps+sq+3rp+3rq)*(ps+sq+rp-rq)\big)^k+$$

$$\big(-(2rq+rp-ps)*(2sq+3rp+ps)\big)^k+$$

$$\big((2rp+rq+sq)*(-2ps+3r*q-sq)\big)^k]$$

For p=2,q=1,r=3,s=1 we get:

2*(2548)^k=

$$13^k*28^k*(3,5,8)^k = 7^k*28^k*(7,8,15)^k = 28^k*(19,80,99)^k = 28^k*(11,85,96)^k =$$

$$13^k*7^k*(32,20,12)^k = 13^k*(0,196,196)^k = 13^k*(156,220,64)^k$$

Taxicab parametric equation for 4-2- 3 with (Ten) chain

$$2[(p^2+pq+q^2)^k(r^2+rs+s^2)^k(3r^2+s^2)^k*(3p^2+q^2)^k=$$

$$(r^2+rs+s^2)^k*(3r^2+s^2)^k*(3p^2+q^2)^k*((p^2-q^2)^k+(q^2+2pq)^k+(-p^2-2pq)^k)$$

$$=$$

$$(p^2+pq+q^2)^k*(3r^2+s^2)^k*(3p^2+q^2)^k*((r^2-s^2)^k+(s^2+2rs)^k+(-r^2-2rs)^k)=$$

$$(3r^2+s^2)^k*(3p^2+q^2)^k[\big((2qr+pr+qs-ps)(pr+qs+ps)\big)^k+\big((2qs+ps+2pr+qr)(ps-qr)\big)^k+\big(-(qs+2ps+pr-qr)(pr+qr+qs)\big)^k]=$$

$$(3r^2+s^2)^k*(3p^2+q^2)^k[\big(-(2pr+qr+ps-qs)(qr+ps+qs)\big)^k+\big((2qr+pr+2ps+qs)(pr-qs)\big)^k+\big((ps+2qs-pr+qr)(ps+pr+qr)\big)^k]=$$

$$(p^2+pq+q^2)^k*(r^2+rs+s^2)*(3p^2+q^2)^k$$
$$*((3r^2+2rs-s^2)^k+(-3r^2+2rs+s^2)^k+(-4r*s)^k)$$

$$=(s^2+r^2+rs)^k*(3p^2+q^2)^k[\big(-(ps-sq+3rp+3rq)(-ps-sq+rp-rq)\big)^k$$

$$+\big(-(2rp+rq-sq)*(2ps+3rq+sq)\big)^k$$

$$+\big((2rq+rp+ps)*(-2sq+3rp-ps)\big)^k$$

$$=(s^2+r^2+rs)^k*(3p^2+q^2)^k[\big((-ps+sq+3rp+3rq)*(ps+sq+rp-rq)\big)^k+$$

$$\big(-(2rq+rp-ps)*(2sq+3rp+ps)\big)^k+$$

$$\big((2rp+rq+sq)*(-2ps+3r*q-sq)\big)^k]$$

$$For\ p=2,q=1,r=3,s=2\ we\ get,for\ k=2,4$$

$$2(7 * 13 * 19 * 31)^k = 2(53599)^k$$

$$19^k * 31^k * 13^k * (3,5,8)^k = 13^k * 7^k * 31^k * (5,16,21)^k = 31^k * 13^k * (153,88,65)^k$$
$$= 31^k * 13^k * (120,23,143)^k =$$

$$19^k * 13^k * 7^k * (35,11,24)^k = 13^k * 19^k * (87,247,160)^k = 13^k * 19^k * (225,208,17)^k$$
$$= 7^k * 31^k * 19^k * (7,8,15)^k = 7^k * 31^k * (280,93,187)^k$$
$$= 7^k * 31^k * (203,275,72)^k$$

$$2[(p^2+pq+q^2)^k(r^2+rs+s^2)^k(3r^2+s^2)^k*(3p^2+q^2)^k=$$

$$(p^2+pq+q^2)^k*(r^2+rs+s^2)*(3r^2+s^2)^k*((3p^2+2pq-q^2)^k+(-3p^2+2pq+q^2)^k+(-4pq)^k)=$$

$$(p^2+pq+q^2)^k*(3r^2+s^2)^k*(3p^2+q^2)^k*((r^2-s^2)^k+(s^2+2rs)^k+(-r^2-2rs)^k)=$$

$$\begin{aligned}(p^2+pq+q^2)^k*(3r^2+s^2)^k*[&\left(-(rq-qs+3pr+3ps)(-rq-qs+pr-ps)\right)^k\\&+\left(-(2pr+ps-qs)(2rq+3ps+qs)\right)^k\\&+\left((2ps+pr+rq)(-2qs+3pr-rq)\right)^k=\end{aligned}$$

$$\begin{aligned}(p^2+pq+q^2)^k*(3r^2+s^2)^k&\\&*\left[\left((-rq+qs+3pr+3ps)(rq+qs+pr-ps)\right)^k\right.\\&+\left(-(2ps+pr-rq)(2qs+3pr+rq)\right)^k\\&\left.+\left((2pr+ps+qs)(-2rq+3ps-qs)\right)^k\right]\end{aligned}$$